

(30) 指數練習題

- $(3^{-\frac{1}{2}})^4 = 3^{-\frac{1}{2} \times 4} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
- $(3^{\frac{5}{3}})^{\frac{2}{5}} = 3^{\frac{5}{3} \times \frac{2}{5}} = 3^{\frac{2}{3}} = \sqrt[3]{3^2} = \sqrt[3]{9}$
- $5^{\frac{3}{2}} \times 5 = 5^{\frac{3}{2}+1} = 5^{\frac{5}{2}} = 5 \cdot 5^{\frac{5-2}{2}} = 5 \cdot 5 \times 5^{\frac{1}{2}} = 25\sqrt{5}$
- $4^{\frac{3}{2}} \div 4^{\frac{1}{2}} = 4^{\frac{3}{2}-\frac{1}{2}} = 4^{\frac{2}{2}} = 4^1 = 4$
- $(3^{\frac{1}{2}})^3 = 3^{\frac{3}{2}} = 3 \times 3^{\frac{3-2}{2}} = 3 \times 3^{\frac{1}{2}} = 3\sqrt{3}$
- $(4^{\frac{1}{2}})^{-1} = 4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{2}$
- $(4^{\frac{1}{3}})(4^{\frac{2}{3}}) = 4^{\frac{1}{3}+\frac{2}{3}} = 4^{\frac{3}{3}} = 4^1 = 4$
- $2^{\frac{1}{2}} \times 3^{\frac{1}{2}} = (2 \times 3)^{\frac{1}{2}} = 6^{\frac{1}{2}} = \sqrt{6}$
- $3^{\frac{3}{2}} \times 4^{\frac{3}{2}} = 12^{\frac{3}{2}} = 12 \cdot 12^{\frac{3-1}{2}} = 12 \cdot 12^{\frac{1}{2}} = 12\sqrt{12} = 12 \times \sqrt{4 \times 3} = 12 \times 2\sqrt{3} = 24\sqrt{3}$
- $(-2)^{\frac{1}{2}} \times (-3)^{\frac{1}{2}} = (6)^{\frac{1}{2}} = \sqrt{6}$
- $3^{\frac{1}{3}} \times 9^{\frac{1}{3}} = (3 \times 9)^{\frac{1}{3}} = (27)^{\frac{1}{3}} = 3$
- $125^{\frac{1}{3}} \times 5 = \sqrt[3]{125} \times 5 = 5 \times 5 = 25$
- $2^{\frac{3}{2}} \times 4^{\frac{3}{2}} = 8^{\frac{3}{2}} = 8 \times 8^{\frac{3-2}{2}} = 8 \times 8^{\frac{1}{2}} = 8\sqrt{8} = 8\sqrt{4 \times 2} = 8 \times 2\sqrt{2} = 16\sqrt{2}$
- $5^{\frac{3}{2}} \times (\frac{2}{5})^{\frac{3}{2}} = (5 \times \frac{2}{5})^{\frac{3}{2}} = 2^{\frac{3}{2}} = 2 \times 2^{\frac{1}{2}} = 2\sqrt{2}$
- $(\frac{1}{5})^{\frac{1}{2}} \times (125)^{\frac{1}{2}} = (\frac{1}{5} \times 125)^{\frac{1}{2}} = (25)^{\frac{1}{2}} = 5$
- $(\frac{1}{5})^{\frac{3}{2}} \times (\frac{1}{4})^{\frac{3}{2}} = (\frac{1}{20})^{\frac{3}{2}} = \frac{1}{(20^2)^{\frac{3}{2}}} = \frac{1}{20 \times 20^{\frac{3-2}{2}}} = \frac{1}{20 \times \sqrt{20}} = \frac{1}{20 \times \sqrt{4 \times 5}} = \frac{1}{20 \times 2\sqrt{5}} = \frac{1}{40\sqrt{5}}$
- $3^{\frac{1}{2}} \times (27)^{\frac{1}{2}} = (3 \times 27)^{\frac{1}{2}} = (81)^{\frac{1}{2}} = 9$

$$18. (5^{\frac{3}{5}})^{\frac{10}{3}} \times 5^{-2} = 5^{\frac{3}{5} \times \frac{10}{3}} \times 5^{-2} = 5^2 \times 5^{-2} = 5^0 = 1$$

$$19. \frac{a^2 b^{\frac{1}{2}}}{ab} = ab^{\frac{1}{2}-1} = ab^{-\frac{1}{2}} = \frac{a}{\sqrt{b}}$$

$$20. (a+b)^{-2} = \frac{1}{(a+b)^2}$$

$$21. (a+b)^{-2} c^2 = \frac{c^2}{(a+b)^2} = \left(\frac{c}{a+b}\right)^2$$

$$22. (a - a^{-1})a = a^2 - 1$$

$$23. (a + a^{-1})(a - a^{-1}) = a^2 - (a^{-1})^2 = a^2 - a^{-2} = a^2 - \frac{1}{a^2} = \frac{a^4 - 1}{a^2}$$

$$24. (a + a^{-1})(a)(a - 1)^{-1} = (a \times a + a^{-1} \times a)(a - 1)^{-1} = (a^2 - 1)(a - 1)^{-1} = (a + 1)(a - 1)(a - 1)^{-1} = (a + 1)(a - 1)^{1-1} = (a + 1)$$

$$25. (a^2 - b^2)(a + b)^{-1} = (a + b)(a - b)(a + b)^{-1} = (a + b)(a + b)^{-1}(a - b) = (a + b)^{1-1}(a - b) = (a - b)$$

$$26. (a^{\frac{1}{2}} + a^{\frac{3}{2}})a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} + a^{\frac{3}{2}+\frac{1}{2}} = a + a^2 = a(1 + a)$$

$$27. a^{\frac{3}{2}}b^{-\frac{1}{2}} + a^{\frac{1}{2}}b^{\frac{1}{2}} = a^{\frac{1}{2}}b^{-\frac{1}{2}}(a + b)$$

$$28. a^{\frac{3}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{\frac{3}{2}} = a^{\frac{1}{2}}b^{\frac{1}{2}}(a + b)$$

$$29. (a^{\frac{1}{2}} + b^{\frac{1}{2}})(a - b) = a^{\frac{1}{2}}a - a^{\frac{1}{2}}b + b^{\frac{1}{2}}a - b^{\frac{1}{2}}b = a^{\frac{3}{2}} - a^{\frac{1}{2}}b + b^{\frac{1}{2}}a - b^{\frac{3}{2}}$$

$$30. a^{-1}b^{-2} + a^{-2}b^{-1} = a^{-2}b^{-2}(a + b) = \frac{a+b}{a^2b^2}$$

31. 解方程式

$$x^2 - (\sqrt{a} + \sqrt{b})x + \sqrt{ab} = 0$$

$$(x - \sqrt{a})(x - \sqrt{b}) = 0$$

$$\therefore x = \sqrt{a}$$

$$x = \sqrt{b}$$

32. 解方程式

$$\sqrt{a}x + \sqrt{b}x = \sqrt{c}$$

$$x(\sqrt{a} + \sqrt{b}) = \sqrt{c}$$

$$x = \frac{\sqrt{c}}{\sqrt{a} + \sqrt{b}} = \frac{\sqrt{c}(\sqrt{a} - \sqrt{b})}{(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})} = \frac{\sqrt{ac} - \sqrt{bc}}{a - b}$$

$$33. x^2 - 3\sqrt{2}x + 4 = 0$$

$$(x - 2\sqrt{2})(x - \sqrt{2}) = 0$$

$$\therefore x = 2\sqrt{2}$$

$$x = \sqrt{2}$$

也可以代公式 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$a=1$$

$$b = -3\sqrt{2}$$

$$c=4$$

$$\therefore x = \frac{3\sqrt{2} \pm \sqrt{(-3\sqrt{2})^2 - 4(1)(4)}}{2} = \frac{3\sqrt{2} \pm \sqrt{18 - 16}}{2} = \frac{3\sqrt{2} \pm \sqrt{2}}{2}$$

$$x = \frac{3\sqrt{2} + \sqrt{2}}{2} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

$$x = \frac{3\sqrt{2} - \sqrt{2}}{2} = \frac{2\sqrt{2}}{2} \sqrt{2}$$

$$34. x^2 - (a^{-1} + 2a)x + 2 = 0$$

$$(x - a^{-1})(x - 2a) = 0$$

$$x = a^{-1} = \frac{1}{a}$$

$$x = 2a$$

$$35. x^2 - (a^{-1} + a)x + 1 = 0$$

$$(x - a^{-1})(x - a) = 0$$

$$x = a^{-1} = \frac{1}{a}$$

$$x = a$$

$$36. 2x^2 + 2\sqrt{2}x + 1 = 0$$

$$(\sqrt{2}x + 1)^2 = 0$$

$$\sqrt{2}x + 1 = 0$$

$$x = -\frac{1}{\sqrt{2}}$$

$$37. x^2 + (\sqrt{5} - \sqrt{2})x + \sqrt{10} = 0$$

$$(x + \sqrt{5})(x - \sqrt{2}) = 0$$

$$(x + \sqrt{5}) = 0$$

$$\therefore x = -\sqrt{5}$$

$$(x - \sqrt{2}) = 0$$

$$\therefore x = \sqrt{2}$$