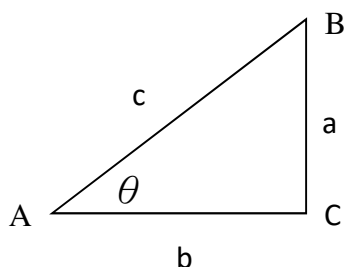


(21) 三角函數的公式一覽表

基本公式



銳角

$$1. \sin \theta = \frac{a}{c} = \frac{a}{\sqrt{a^2+b^2}}$$

$$2. \cos \theta = \frac{b}{c} = \frac{b}{\sqrt{a^2+b^2}}$$

$$3. \tan \theta = \frac{a}{b}$$

$$4. \csc \theta = \frac{1}{\sin \theta} = \frac{\sqrt{a^2+b^2}}{a}$$

$$5. \sec \theta = \frac{1}{\cos \theta} = \frac{\sqrt{a^2+b^2}}{b}$$

$$6. \cot \theta = \frac{1}{\tan \theta} = \frac{b}{a}$$

$$7. a = c \sin \theta$$

$$8. b = c \cos \theta$$

廣義角

$$9. \sin(2n\pi \pm \theta) = \pm \sin \theta$$

$$10. \cos(2n\pi \pm \theta) = \pm \cos \theta$$

$$11. \tan(n\pi \pm \theta) = \pm \tan \theta$$

$$12. \cot(n\pi \pm \theta) = \pm \cot \theta$$

$$13. \sec(2n\pi \pm \theta) = \pm \sec \theta$$

$$14. \csc(2n\pi \pm \theta) = \pm \csc \theta$$

三角函數在各象限的正或負

	第一象限	第二象限	第三象限	第四象限
$\sin \theta$	+	+	-	-
$\cos \theta$	+	-	-	+
$\tan \theta$	+	-	+	-

$$15. \sin\left(\frac{n\pi}{2} \pm \theta\right) = \cos \theta \quad (n=4k+1, k \in \mathbb{Z});$$

$$\sin\left(\frac{n\pi}{2} \pm \theta\right) = -\cos \theta \quad (n=4k+3, k \in \mathbb{Z})$$

$$16. \cos\left(\frac{n\pi}{2} \pm \theta\right) = \mp \sin \theta \quad (n=4k+1, k \in \mathbb{Z});$$

$$\cos\left(\frac{n\pi}{2} \pm \theta\right) = \pm \sin \theta \quad (n=4k+3, k \in \mathbb{Z})$$

$$17. \tan\left(\frac{n\pi}{2} \pm \theta\right) = \mp \cot \theta$$

$$18. \cot\left(\frac{n\pi}{2} \pm \theta\right) = \mp \tan \theta$$

$$19. \csc\left(\frac{n\pi}{2} \pm \theta\right) = \sec \theta \quad (n=4k+1, k \in \mathbb{Z});$$

$$\csc\left(\frac{n\pi}{2} \pm \theta\right) = -\sec \theta \quad (n=4k+3, k \in \mathbb{Z})$$

$$20. \sec\left(\frac{n\pi}{2} \pm \theta\right) = \mp \csc \theta \quad (n=4k+1, k \in \mathbb{Z});$$

$$\sec\left(\frac{n\pi}{2} \pm \theta\right) = \pm \csc \theta \quad (n=4k+3, k \in \mathbb{Z})$$

特例角的三角函數值

	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

$$21. \sin^2 \theta + \cos^2 \theta = 1$$

$$22. 1 + \tan^2 \theta = \sec^2 \theta$$

$$23. 1 + \cot^2 \theta = \csc^2 \theta$$

$$24. \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$25. \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$25-1. \sin(-\theta) = -\sin \theta$$

$$25-2. \cos(-\theta) = \cos \theta$$

$$25-3. \tan(-\theta) = -\tan \theta$$

$$25-4. \cot(-\theta) = -\cot \theta$$

$$25-5. \sec(-\theta) = \sec \theta$$

$$25-6. \csc(-\theta) = -\csc \theta$$

正弦定理(若 a 、 b 、 c 為 $\triangle ABC$ 的三邊長)

$$26. \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

R 是 $\triangle ABC$ 外接圓的半徑

餘弦定理(若 a 、 b 、 c 為 $\triangle ABC$ 的三邊長)

$$27. a^2 = b^2 + c^2 - 2bc \cos A$$

$$28. b^2 = a^2 + c^2 - 2ac \cos B$$

$$29. c^2 = a^2 + b^2 - 2ab \cos C$$

$$30. \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$31. \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$32. \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

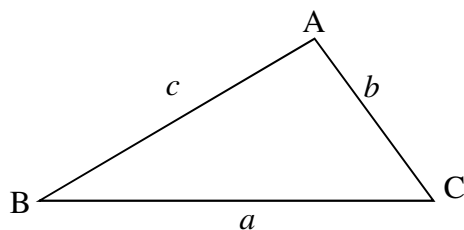
投影定理

$$33. a = b \cos C + c \cos B$$

$$34. b = a \cos C + c \cos A$$

$$35. c = b \cos A + a \cos B$$

加法定理



$$36. \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$37. \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$38. \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$39. \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$40. \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$41. \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$42. \cot(\alpha + \beta) = \frac{\cot \alpha \cot \beta - 1}{\cot \beta + \cot \alpha}$$

$$43. \cot(\alpha - \beta) = \frac{\cot \alpha \cot \beta + 1}{\cot \beta - \cot \alpha}$$

$$44. \sec(\alpha + \beta) = \frac{\sec \alpha \sec \beta}{1 - \tan \alpha \tan \beta}$$

$$45. \sec(\alpha - \beta) = \frac{\sec \alpha \sec \beta}{1 + \tan \alpha \tan \beta}$$

$$46. \csc(\alpha + \beta) = \frac{\csc \alpha \csc \beta}{\cot \alpha + \cot \beta - 1}$$

$$47. \csc(\alpha - \beta) = \frac{\csc \alpha \csc \beta}{\cot \alpha \cot \beta + 1}$$

倍角定理

$$48. \sin 2\theta = 2 \sin \theta \cos \theta$$

$$49. \cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$50. \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$51. \cot 2\theta = \frac{\cot \theta - \tan \theta}{2} = \frac{\cot^2 \theta - 1}{2 \cot \theta}$$

$$52. \sec 2\theta = \frac{\sec^2 \theta}{1 - \tan^2 \theta} = \frac{\cot \theta + \tan \theta}{\cot \theta - \tan \theta}$$

$$53. \csc 2\theta = \frac{\sec \theta \csc \theta}{2} = \frac{\cot \theta + \tan \theta}{2}$$

$$54. \sin 3\theta = -4 \sin^3 \theta + 3 \sin \theta$$

$$55. \cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$56. \tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

$$57. \cot 3\theta = \frac{\cot^3 \theta - 3 \cot \theta}{3 \cot^2 \theta - 1}$$

$$58. \sec 3\theta = \frac{\sec^3 \theta}{4 - 3 \sec^2 \theta}$$

$$59. \csc 3\theta = \frac{\csc^3 \theta}{3 \csc^2 \theta - 4}$$

半角公式

$$60. \sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$61. \cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$62. \tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \pm \frac{\sin \theta}{1 + \cos \theta} = \pm \frac{1 - \cos \theta}{\sin \theta} = \pm (\csc \theta - \cot \theta)$$

$$63. \cot \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} = \pm \frac{\sin \theta}{1 - \cos \theta} = \pm \frac{1 + \cos \theta}{\sin \theta} = \pm (\csc \theta + \cot \theta)$$

積化和差公式

$$64. \sin \alpha \cos \beta = \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{2}$$

$$65. \cos \alpha \sin \beta = \frac{\sin(\alpha + \beta) - \sin(\alpha - \beta)}{2}$$

$$66. \cos \alpha \cos \beta = \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2}$$

$$67. \sin \alpha \sin \beta = \frac{-\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2}$$

和差化積公式

$$68. \sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$69. \sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$70. \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$71. \cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

