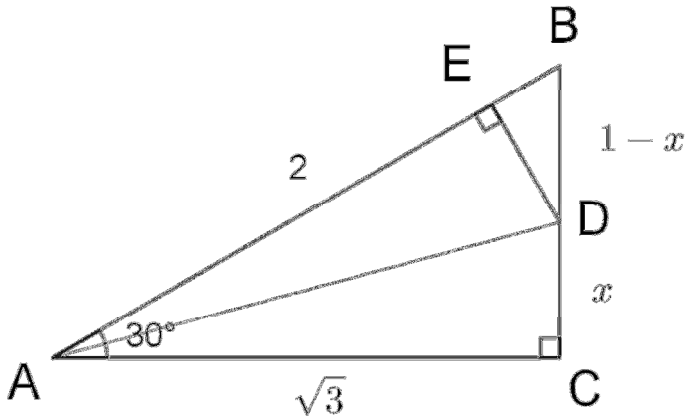


(14) 求 $\sin 15^\circ$



AB 為 $\angle A$ 的分角線

因為 $\sin 30^\circ = \frac{1}{2}$ ，故令 $\overline{BC} = 1$ ， $\overline{AB} = 2$ ，可得 $\overline{AC} = \sqrt{3}$

令 $\overline{DC} = x$ ，令 $\overline{DE} \perp \overline{AB}$

$\because \square ADE \cong \triangle ADC \therefore \overline{DE} = x$

$$\overline{BD} = 1 - x$$

$$\overline{BE} = 2 - \overline{AE} = 2 - \overline{AC} = 2 - \sqrt{3}$$

$$\sin B = \sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{\overline{DE}}{\overline{BD}} = \frac{x}{1 - x}$$

$$2x = \sqrt{3} - \sqrt{3}x$$

$$x = \frac{\sqrt{3}}{2 + \sqrt{3}} = \frac{\sqrt{3}(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} = \sqrt{3}(2 - \sqrt{3})$$

$$x = 2\sqrt{3} - 3$$

$$\overline{AD}^2 = x^2 + (\sqrt{3})^2$$

$$= (2\sqrt{3} - 3)^2 + 3$$

$$= 24 - 12\sqrt{3}$$

$$= 12(2 - \sqrt{3})$$

$$\therefore \sin 15^\circ = \frac{\overline{DC}}{\overline{AD}}$$

$$\begin{aligned}\therefore (\sin 15^\circ)^2 &= \frac{\overline{DC}^2}{\overline{AD}^2} = \frac{(2\sqrt{3} - 3)^2}{12(2 - \sqrt{3})} \\ &= \frac{(2\sqrt{3} - 3)^2 \sqrt{3}}{12(2\sqrt{3} - 3)} \\ &= \frac{\sqrt{3}(2\sqrt{3} - 3)}{12} \\ &= \frac{6 - 3\sqrt{3}}{12} \\ &= \frac{2 - \sqrt{3}}{4} \dots\dots\dots (1)\end{aligned}$$

$$\therefore \sin 15^\circ = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$\text{令 } 2 - \sqrt{3} = (x - y)^2$$

$$x^2 + y^2 - 2xy = 2 - \sqrt{3}$$

$$x^2 + y^2 = 2$$

$$-2xy = -\sqrt{3}$$

$$\text{得 } x^2 + y^2 = 2$$

$$xy = \frac{\sqrt{3}}{2}$$

$$x = \frac{\sqrt{3}}{2y}$$

$$\text{得 } \frac{3}{4y^2} + y^2 = 2$$

$$\text{得 } 3 + 4y^4 = 8y^2$$

$$\text{得 } 4y^4 - 8y^2 + 3 = 0$$

$$(2y^2 - 3)(2y^2 - 1) = 0$$

$$\therefore y^2 = \frac{3}{2}$$

$$\text{或 } y^2 = \frac{1}{2}$$

$$\therefore y = \frac{\sqrt{3}}{\sqrt{2}} \text{ 或 } y = \frac{1}{\sqrt{2}}$$

$$\text{可得 } x = \frac{1}{\sqrt{2}} \text{ 或 } x = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\text{可得 } \sqrt{2 - \sqrt{3}} = \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

$$\text{代入(1)得 } \sin 15^\circ = \frac{\frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{2} = \frac{\frac{2\sqrt{3} - 2}{\sqrt{2} - \sqrt{2}}}{2 \times 2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$