(02) 根式運算

一、基本運算規則

1.

$\sqrt{2} \times \sqrt{3}$

$= \sqrt{2 \times 3}$

$= \sqrt{6}$

2.

$\sqrt{2} \times \sqrt{5}$

$= \sqrt{2 \times 5}$

$= \sqrt{10}$

3.

$\sqrt{2} \times (\sqrt{3} + \sqrt{2})$

$= \sqrt{2 \times 3} + \sqrt{2 \times 2}$

$= \sqrt{6} + 2$

4.

$\sqrt{3} \times (\sqrt{5} - \sqrt{2})$

$= \sqrt{3 \times 5} - \sqrt{3 \times 2}$

$= \sqrt{15} - \sqrt{6}$

5.

$\sqrt{2} \times (\sqrt{3} + \sqrt{6})$

$= \sqrt{2 \times 3} + \sqrt{2 \times 6}$

$= \sqrt{6} + \sqrt{12}$

6.

$\sqrt{5} \times (\sqrt{2} - \sqrt{3})$

$= \sqrt{5 \times 2} - \sqrt{5 \times 3}$

$= \sqrt{10} - \sqrt{15}$

二、利用乘法公式乘開根式

7.

$(\sqrt{6}+\sqrt{2})(\sqrt{3}+\sqrt{5})$

$=\sqrt{6\times 3}+\sqrt{6\times 5}$

$+\sqrt{2\times 3}+\sqrt{2\times 5}$

$=\sqrt{18}+\sqrt{30}+\sqrt{6}+\sqrt{10}$

8.

$(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$

$=(\sqrt{3})^2-(\sqrt{2})^2$

$=3-2$

$=1$

9.

$(\sqrt{3}+1)(\sqrt{3}-1)$

$=(\sqrt{3})^2-(1)^2$

$=3-1$

$=2$

10.

$(\sqrt{2}+1)(\sqrt{2}-1)$

$=(\sqrt{2})^2-(1)^2$

$=2-1$

$=1$

11.

$(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$

$=(\sqrt{5})^2-(\sqrt{3})^2$

$=5-3$

$=2$

三、提出整數

12.

$\sqrt{12}$

$= \sqrt{4 \times 3}$

$= \sqrt{2^2 \times 3}$

$= 2\sqrt{3}$

13.

$\sqrt{32}$

$= \sqrt{16 \times 2}$

$= \sqrt{4^2 \times 2}$

$= 4\sqrt{2}$

14.

$\sqrt{45}$

$= \sqrt{9 \times 5}$

$= \sqrt{3^2 \times 5}$

$= 3\sqrt{5}$

15.

$\sqrt{50}$

$= \sqrt{25 \times 2}$

$= \sqrt{5^2 \times 2}$

$= 5\sqrt{2}$

16.

$\sqrt{96}$

$= \sqrt{16 \times 6}$

$= \sqrt{4^2 \times 6}$

$= 4\sqrt{6}$

17.

$\sqrt{2} \times \sqrt{6}$

$= \sqrt{12}$

$= \sqrt{4 \times 3}$

$= \sqrt{2^2 \times 3}$

$= 2\sqrt{3}$

18.

$\sqrt{6} \times \sqrt{3}$

$= \sqrt{18}$

$= \sqrt{9 \times 2}$

$= \sqrt{3^2 \times 2}$

$= 3\sqrt{2}$

19.

$\sqrt{2} \times \sqrt{12}$

$= \sqrt{24}$

$= \sqrt{4 \times 6}$

$= \sqrt{2^2 \times 6}$

$= 2\sqrt{6}$

四、根式有理化

20.

$$\frac{1}{\sqrt{2}+1}$$

$$=\frac{1\times(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}$$

$$=\frac{\sqrt{2}-1}{(\sqrt{2})^2-1^2}$$

$$=\frac{\sqrt{2}-1}{2-1}$$

$$=\sqrt{2}-1$$

21.

$$\frac{2}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{2\times(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

$$=\frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$=\frac{2(\sqrt{5}-\sqrt{3})}{5-3}$$

$$=\frac{2(\sqrt{5}-\sqrt{3})}{2}$$

$$=\sqrt{5}-\sqrt{3}$$

22.

$$\frac{2\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$

$$=\frac{2\sqrt{2}\times(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}$$

$$=\frac{2\sqrt{2}(\sqrt{6}-\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2}$$

$$=\frac{2\sqrt{12}-2\times 2}{6-2}$$

$$=\frac{4\sqrt{3}-4}{4}$$

$$=\sqrt{3}-1$$

23.

$$\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{3\sqrt{2}\times(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$=\frac{3\sqrt{2}(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$=\frac{3\sqrt{6}-3\times 2}{3-2}$$

$$=\frac{3\sqrt{6}-6}{1}$$

$$=3(\sqrt{6}-2)$$

五、利用乘法公式乘開根式(2)

24.

25.

$(1+\sqrt{2})^2$

$= 12 \quad + 2\sqrt{2} + (\sqrt{2})^2$

$= 1 + 2\sqrt{2} + 2$

$= 3 + 2\sqrt{2}$

$(\sqrt{3}-\sqrt{2})^2$

$= (\sqrt{3})2 \quad - 2\sqrt{3} \times \sqrt{2} + (\sqrt{2})^2$

$= 3 - 2\sqrt{6} + 2$

$= 5 - 2\sqrt{6}$

六、根式應用

26. 已知 $a^2=4$，求 a。

$a^2=4$，$a=\pm\sqrt{4}=\pm 2$

27. 已知 $a^2=9$，求 a。

$a^2=9$，$a=\pm\sqrt{9}=\pm 3$

28. 已知 $a^2=20$，求 a。

$a^2=20$，$a=\pm\sqrt{20}=\pm 2\sqrt{5}$

29. 已知 $a^2=12$，求 a。

$a^2=12$，$a=\pm\sqrt{12}=\pm 2\sqrt{3}$

30. 已知 $a^2=18$，求 a。

$a^2=18$，$a=\pm\sqrt{18}=\pm 3\sqrt{2}$

31. $a\sqrt{2}+b\sqrt{8}+3b=5\sqrt{2}+6$，求 a、b。

$a\sqrt{2}+b\sqrt{8}+3b$

$=a\sqrt{2}+2b\sqrt{2}+3b$

$=(a+2b)\sqrt{2}+3b$

$=5\sqrt{2}+6$

$\therefore 3b=6$，$b=2$

$a+2b=5$，$a+2\times 2=5$，$a=1$

答案：$a=1$、$b=2$

32. $(3+\sqrt{3})a+4(1+2\sqrt{3}b)=7+9\sqrt{3}$，

求 a、b。

$(3+\sqrt{3})a+4(1+2\sqrt{3}b)$

$=3a+\sqrt{3}a+4+8\sqrt{3}b$

$=3a+4+(a+8b)\sqrt{3}$

$=7+9\sqrt{3}$

∴$3a+4=7$，$a=1$

代入 $a+8b=9$

$1+8b=9$，$b=1$

答案：$a=1$、$b=1$

33. $a\sqrt{2}+2b\sqrt{2}+a+b=8\sqrt{2}+5$，

求 a、b。

$a\sqrt{2}+2b\sqrt{2}+a+b$

$=(a+2b)\sqrt{2}+a+b$

$=8\sqrt{2}+5$

$\therefore\begin{cases}a+2b=8\\a+b=5\end{cases}$

解得 $a=2$、$b=3$

答案：$a=2$、$b=3$

七、根號比大小

34. $2\sqrt{3}, 3\sqrt{2}$比大小

$(2\sqrt{3})^2 = 4 \times 3 = 12$

$(3\sqrt{2})^2 = 9 \times 2 = 18$

12<18

$\therefore 2\sqrt{3} < 3\sqrt{2}$

35. $\sqrt{5}, 2\sqrt{2}$比大小

$(\sqrt{5})^2 = 5$

$(2\sqrt{2})^2 = 4 \times 2 = 8$

5<8

$\therefore \sqrt{5} < 2\sqrt{2}$

36. $4\sqrt{5}, 2\sqrt{21}$比大小

$(4\sqrt{5})^2 = 16 \times 5 = 80$

$(2\sqrt{21})^2 = 4 \times 21 = 84$

80<84

$\therefore 4\sqrt{5} < 2\sqrt{21}$

37. $6\sqrt{2}, 9$比大小

$(6\sqrt{2})^2 = 36 \times 2 = 72$

$(9)^2 = 81$

72<81

$\therefore 6\sqrt{2} < 9$

38. $\frac{\sqrt{2}}{3}, \frac{5}{4}$比大小

通分變成：$\frac{4\sqrt{2}}{12}, \frac{15}{12}$

將分子平方比較

$(4\sqrt{2})^2 = 16 \times 2 = 32$

$(15)^2 = 225$

32<225

$\therefore \frac{\sqrt{2}}{3} < \frac{5}{4}$

39. $\frac{5\sqrt{2}}{4}, \frac{2\sqrt{7}}{3}$比大小

通分變成：$\frac{15\sqrt{2}}{12}, \frac{8\sqrt{7}}{12}$

將分子平方比較

$(15\sqrt{2})^2 = 225 \times 2 = 450$

$(8\sqrt{7})^2 = 64 \times 7 = 448$

450>448

$\therefore \frac{5\sqrt{2}}{4} > \frac{2\sqrt{7}}{3}$

40. $\frac{2\sqrt{3}}{5}, \frac{\sqrt{2}}{3}$比大小

通分變成：$\frac{6\sqrt{3}}{15}, \frac{5\sqrt{2}}{15}$

將分子平方比較

$(6\sqrt{3})^2 = 36 \times 3 = 108$

$(5\sqrt{2})^2 = 25 \times 2 = 50$

108>50

$\therefore \frac{2\sqrt{3}}{5} > \frac{\sqrt{2}}{3}$