

(1) 符號運算

數學符號運算

$$\begin{aligned} 1. \quad & \frac{1}{a} + \frac{1}{b} \\ &= \frac{b}{ab} + \frac{a}{ab} \\ &= \frac{a+b}{ab} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{1}{2a} - \frac{2}{3b} \\ &= \frac{3b}{6ab} - \frac{4a}{6ab} \\ &= \frac{3b-4a}{6ab} \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{3}{4a} - \frac{1}{6b} \\ &= \frac{9b}{12ab} - \frac{2a}{12ab} \\ &= \frac{9b-2a}{12ab} \end{aligned}$$

$$\begin{aligned} 4. \quad & \frac{1}{ab} + \frac{1}{c} \\ &= \frac{c}{abc} + \frac{ab}{abc} \\ &= \frac{c+ab}{abc} \end{aligned}$$

$$\begin{aligned} 5. \quad & \frac{a}{b} + \frac{c}{d} \\ &= \frac{ad}{bd} + \frac{bc}{bd} \\ &= \frac{ad+bc}{bd} \end{aligned}$$

$$\begin{aligned} 6. \quad & a + \frac{1}{a} \\ &= \frac{a^2}{a} + \frac{1}{a} \\ &= \frac{a^2+1}{a} \end{aligned}$$

$$\begin{aligned} 7. \quad & \frac{1}{a} + \frac{2}{a^2} \\ &= \frac{a}{a^2} + \frac{2}{a^2} \\ &= \frac{a+2}{a^2} \end{aligned}$$

$$\begin{aligned} 8. \quad & \frac{1}{a+b} + \frac{1}{a-b} \\ &= \frac{a-b}{(a+b)(a-b)} + \frac{a+b}{(a-b)(a+b)} \\ &= \frac{(a-b) + (a+b)}{a^2 - b^2} \\ &= \frac{2a}{a^2 - b^2} \end{aligned}$$

$$\begin{aligned} 9. \quad & \frac{1}{\sqrt{a} + \sqrt{b}} - \frac{1}{\sqrt{a} - \sqrt{b}} \\ &= \frac{\sqrt{a} - \sqrt{b}}{(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})} - \frac{\sqrt{a} + \sqrt{b}}{(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})} \end{aligned}$$

$$= \frac{(\sqrt{a} - \sqrt{b}) - (\sqrt{a} + \sqrt{b})}{a - b}$$

$$= \frac{-2\sqrt{b}}{a - b}$$

$$10. \frac{1}{a^2} + \frac{1}{b^2}$$

$$= \frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2}$$

$$= \frac{b^2 + a^2}{a^2b^2}$$

$$11. \frac{1}{a^2} - \frac{1}{b^2}$$

$$= \frac{b^2}{a^2b^2} - \frac{a^2}{a^2b^2}$$

$$= \frac{b^2 - a^2}{a^2b^2}$$

$$= \frac{(b + a)(b - a)}{a^2b^2}$$

$$12. \frac{1}{a} \times \frac{1}{b}$$

$$= \frac{1}{ab}$$

$$13. \frac{a}{b} \times \frac{1}{a^2}$$

$$= \frac{1}{ab}$$

$$14. \frac{1}{a} \div \frac{1}{b}$$

$$= \frac{1}{a} \times \frac{b}{1}$$

$$= \frac{b}{a}$$

$$15. \frac{a^2}{\frac{1}{b}}$$

$$= a^2 \div \frac{1}{b}$$

$$= a^2 \times \frac{b}{1}$$

$$= ab$$

$$16. \frac{ab}{c} \times \frac{2}{b}$$

$$= \frac{2a}{c}$$

$$17. (a^2 - b^2) \times \frac{1}{a - b}$$

$$= \frac{(a + b)(a - b)}{a - b}$$

$$= a + b$$

$$18. \frac{\frac{a}{b}}{\frac{c}{d}}$$

$$= \frac{a}{b} \div \frac{c}{d}$$

$$19. \frac{a^2}{\frac{b^2}{a^2}}$$

$$= a^2 \div \frac{b^2}{a^2}$$

$$\begin{aligned} &= \frac{a}{b} \times \frac{d}{c} \\ &= \frac{ad}{bc} \end{aligned}$$

$$\begin{aligned} &= a^2 \times \frac{a^2}{b^2} \\ &= \frac{a^4}{b^2} \end{aligned}$$

20. $\frac{a-b}{a^2-b^2}$

$$\begin{aligned} &= \frac{a-b}{(a+b)(a-b)} \\ &= \frac{1}{a+b} \end{aligned}$$

$$\begin{aligned}
21. \quad & (-a) + b + 3a - 2b \\
& = (-a + 3a) + (b - 2b) \\
& = 2a - b
\end{aligned}$$

$$\begin{aligned}
22. \quad & -a(a + b) \\
& = -a^2 + ab
\end{aligned}$$

$$\begin{aligned}
23. \quad & a(a - b) - 2a(a + b) \\
& = a^2 - ab - 2a^2 - 2ab \\
& = (a^2 - 2a^2) - ab - 2ab \\
& = (-a^2) - 3ab
\end{aligned}$$

$$\begin{aligned}
24. \quad & a(a + b) - c(d - a) \\
& = a^2 + ab - cd + ca
\end{aligned}$$

$$\begin{aligned}
25. \quad & (a + b)(c + d) \\
& = a(c + d) + b(c + d) \\
& = ac + ad + bc + bd
\end{aligned}$$

$$\begin{aligned}
26. \quad & 3a(c + d) - 4c(a - d) \\
& = 3ac + 3ad - 4ac + 4cd \\
& = (3ac - 4ac) + 3ad + 4cd \\
& = (-ac) + 3ad + 4cd
\end{aligned}$$

$$\begin{aligned}
27. \quad & 3a(a + b) - 4a(a - b) \\
& = 3a^2 + 3ab - 4a^2 + 4ab \\
& = (3a^2 - 4a^2) + (3ab + 4ab) \\
& = (-a^2) + 7ab
\end{aligned}$$

$$\begin{aligned}
28. \quad & a(a + 2b) - 2a(2a - b) \\
& = a^2 + 2ab - 4a^2 + 2ab \\
& = -3a^2 + 4ab
\end{aligned}$$

$$\begin{aligned}
29. \quad & -a(b - c) + b(a - c) \\
& = -ab + ac + ab - bc \\
& = (-ab + ab) + ac - bc \\
& = ac - bc
\end{aligned}$$

$$\begin{aligned}
30. \quad & 4a(a + 2b) - 3a(a - b) \\
& = 4a^2 + 8ab - 3a^2 + 3ab \\
& = (4a^2 - 3a^2) + (8ab + 3ab) \\
& = a^2 + 11ab
\end{aligned}$$

$$\begin{aligned}
31. \quad & x - a = b \\
& x = a + b
\end{aligned}$$

$$\begin{aligned}
32. \quad & 3x - a = 4x + b \\
& 3x - 4x = a + b \\
& -x = a + b \\
& x = -(a + b)
\end{aligned}$$

$$\begin{aligned}
33. \quad & \frac{x - a}{2x - b} = 1 \\
& x - a = 2x - b \\
& x - 2x = a - b \\
& -x = a - b \\
& x = -a + b
\end{aligned}$$

$$\begin{aligned}
34. \quad & \frac{x}{3a} + a = b \\
& \frac{x}{3a} = b - a \\
& x = 3a(b - a)
\end{aligned}$$

$$\begin{aligned}
35. \quad & \frac{2x - a}{x + a} = \frac{1}{2} \\
& 2(2x - a) = x + a \\
& 4x - 2a = x + a
\end{aligned}$$

$$\begin{aligned}
36. \quad & 2x - b = 3a + a \\
& 2x - 3x = a + b \\
& -x = a + b \\
& x = -(a + b)
\end{aligned}$$

$$4x - x = a + 2a$$

$$3x = 3a$$

$$x = a$$

$$\begin{aligned} 37. \quad \frac{x - 2a}{x + 2a} &= \frac{2a}{3} \\ 3(x - 2a) &= 2a(x + 2a) \\ 3x - 6a &= 2ax + 4a^2 \\ 3x - 2ax &= 4a^2 + 6a \\ x(3 - 2a) &= 4a^2 + 6a \\ x &= \frac{4a^2 + 6a}{3 - 2a} \end{aligned}$$

$$\begin{aligned} 39. \quad 4ax + 5bx &= 3 \\ x(4a + 5b) &= 3 \\ x &= \frac{3}{4a + 5b} \end{aligned}$$

$$\begin{aligned} 38. \quad 2ax + 3 &= 4bx - 7 \\ 2ax - 4bx &= -7 - 3 \\ 2x(a - 2b) &= -10 \\ x(a - 2b) &= -5 \\ x &= \frac{-5}{a - 2b} \end{aligned}$$

$$\begin{aligned} 40. \quad ax + b &= cx + d \\ ax - cx &= d - b \\ x(a - c) &= d - b \\ x &= \frac{d - b}{a - c} \end{aligned}$$

41. 解聯立方程式

$$\begin{cases} ax + by = c \dots (1) \\ x - y = d \dots (2) \end{cases}$$

詳解：

$$(2) \times a \quad ax - ay = ad \dots (3)$$

$$(1) - (3) \quad (b + a)y = c - ad$$

$$\therefore y = \frac{c - ad}{b + a}$$

$$(2) \times b \quad bx - by = bd \dots (4)$$

$$(1) + (4) \quad (a + b)x = c + bd$$

$$\therefore x = \frac{c + bd}{a + b}$$

$$\text{答：} x = \frac{c + bd}{a + b}, y = \frac{c - ad}{b + a}$$

42. 解聯立方程式

$$\begin{cases} ax + y = b \dots (1) \\ ax - y = d \dots (2) \end{cases}$$

詳解：

$$(1) + (2) \quad 2ax = b + d$$

$$\therefore x = \frac{b + d}{2a}$$

$$(1) - (2) \quad 2y = b - d$$

$$\therefore y = \frac{b - d}{2}$$

$$\text{答：} x = \frac{b + d}{2a}, y = \frac{b - d}{2}$$

43. 解聯立方程式

$$\begin{cases} x + ay = b \dots (1) \\ x - ay = c \dots (2) \end{cases}$$

詳解：

$$(1) + (2) \quad 2x = b + c$$

$$\therefore x = \frac{b+c}{2}$$

$$(1) - (2) \quad 2ay = b - c$$

$$\therefore y = \frac{b-c}{2a}$$

$$\text{答：} x = \frac{b+c}{2}, y = \frac{b-c}{2a}$$

44. 解聯立方程式

$$\begin{cases} x + y = a \dots (1) \\ x - y = b \dots (2) \end{cases}$$

詳解：

$$(1) + (2) \quad 2x = a + b$$

$$\therefore x = \frac{a+b}{2}$$

$$(1) - (2) \quad 2y = a - b$$

$$\therefore y = \frac{a-b}{2}$$

$$\text{答：} x = \frac{a+b}{2}, y = \frac{a-b}{2}$$

45. 解聯立方程式

$$\begin{cases} ax + y = b \dots (1) \\ x - ay = c \dots (2) \end{cases}$$

詳解：

$$(2) \times a \quad 2a - a^2y = ac \dots (3)$$

$$(1) - (3)$$

$$y + a^2y = b - ac$$

$$(1 + a^2)y = b - ac$$

$$\therefore y = \frac{b-ac}{1+a^2}$$

$$(1) \times a \quad a^2x + ay = ab \dots (4)$$

$$(2) + (4)$$

$$x + a^2x = c + ab$$

$$(1 + a^2)x = c + ab$$

$$\therefore x = \frac{c+ab}{1+a^2}$$

$$\text{答：} x = \frac{c+ab}{1+a^2}, y = \frac{b-ac}{1+a^2}$$

46. 解一元二次方程式

$$ax^2 + a^2x = 0 \quad (a \neq 0)$$

詳解：

$$ax^2 + a^2x = 0$$

$$ax(x + a) = 0$$

$$\begin{cases} ax = 0, x = 0 \\ x + a = 0, x = -a \end{cases}$$

$$\text{答：} x = 0, -a$$

47. 解一元二次方程式

$$x^2 - 2ax + a^2 = 0$$

48. 解一元二次方程式

$$x^2 - (a + b)x + ab = 0$$

詳解：

$$x^2 - 2ax + a^2 = 0$$

$$(x - a)^2 = 0$$

$$x = a$$

(重根)

答： $x = a$ (重根)

詳解：

$$x^2 - (a + b)x + ab = 0$$

$$(x - a)(x - b) = 0$$

$$x = a, b$$

答： $x = a, b$

49. 解一元二次方程式

$$abx^2 + (a + b)x + 1 = 0$$

詳解：

$$abx^2 + (a + b)x + 1 = 0$$

$$(ax + 1)(bx + 1) = 0$$

$$\begin{cases} ax + 1 = 0, x = -\frac{1}{a} \\ bx + 1 = 0, x = -\frac{1}{b} \end{cases}$$

答： $x = -\frac{1}{a}, -\frac{1}{b}$

50. 解一元二次方程式

$$x^2 - 4ax + 3a^2 = 0$$

詳解：

$$x^2 - 4ax + 3a^2 = 0$$

$$(x - 3a)(x - a) = 0$$

$$\begin{cases} x - 3a = 0, x = 3a \\ x - a = 0, x = a \end{cases}$$

答： $x = 3a, a$